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ON MEAN MERIDIONAL CIRCULATIONS IN THE ATMOSPHERE [1]

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ABSTRACT

The purpose of the study is to make a detailed investigation of mean meridional circulations forced by given eddy transports of heat and momentum and to describe the vertical variation of the energy conversions for the zonally averaged flow.

A nonhomogeneous second-order partial differential equation for the vertical p-velocity, ω, is obtained from the quasi-geostrophic vorticity and thermodynamic equations. The method of separation of variables is used to solve the zonally averaged form of this equation such that zonally averaged vertical p-velocity, ω_z, is expressed as a series of Legendre polynomials. The boundary conditions used are that ω_z is zero at the top of the atmosphere and that at the surface it is equal to that value of ω_z which is produced by the topography of the earth. After the solution for ω_z is obtained, the mean meridional velocity is determined from the zonally averaged continuity equation.

The diabatic heating in the meridional plane is estimated from the zonally averaged steady-state thermodynamic equation. Computations of the zonal available potential energy and the conversion from zonal available potential energy to zonal kinetic energy are made using the distributions of diabatic heating, the vertical p-velocity and the temperature in the meridional plane.

The general conclusions which can be drawn on the basis of the calculations are:

(1) Three-cell meridional circulations are produced by the eddy transport processes in the atmosphere.

(2) The eddy transport of momentum is twice as effective as the eddy transport of heat in forcing the meridional circulations.

(3) The influence of the planetary scale motion on the circulation is predominant during winter whereas that of the baroclinically unstable waves dominates the forcing mechanism during the other seasons.

(4) The seasonal variation of the meridional circulations shows that the circulation cells move toward the pole and undergo a decrease in their intensity from winter to summer.

(5) The net diabatic heating in the meridional plane is positive south of 40° N. and negative north of that latitude during winter months. In the upper troposphere, the heating decreases gradually with height in the region of net heating whereas the cooling decreases sharply in the region of net cooling.

(6) The generation of zonal available potential energy is maximum in the lower troposphere, decreases sharply with height, and becomes negative in the lower stratosphere.

(7) The conversion from zonal available potential energy to zonal kinetic energy is positive in the lower troposphere and negative in the upper troposphere.

1. INTRODUCTION

It is generally recognized that the quantitative estimates of the mean meridional circulations by the so-called direct method suffer large uncertainties, especially in the middle and high latitudes. It is therefore necessary to infer these circulations considering the internal dynamics of the atmosphere. Several recent investigations, Mintz and Lang [14], Gilman [6], and Holopainen [7] have estimated the strength of the mean meridional circulation necessary to balance the angular momentum in steady state. Kuo [10] on the other hand, estimated the strength of these circulations required to balance the heat and momentum in steady state. He showed that the meridional circulations are forced motions and the condition for the free motions are not satisfied for the large-scale motions in the atmosphere. He pointed out that the dominant forcing functions are the derivatives of eddy transfer of zonal momentum and eddy transfer of sensible heat. The quantitative estimates of the circulations were based on the most representative forcing functions.

The present study, in principle, is similar to Kuo's method, but based on a diagnostic model so that the mean

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meridional circulations can be inferred whenever the forcing function is known. In the first part of the study, the variations of meridional circulations are investigated as a function of time and the scales of motion in the atmosphere. The second part describes the vertical variations of the energy conversion for the zonally averaged flow.

2. FORMULATION AND METHOD OF SOLUTION

2.1 FORMULATION

The present attempt of computing the mean meridional circulations is based on a quasi-geostrophic model atmosphere. The simplified form of the vorticity equation which is consistent with the quasi-geostrophic theory and satisfying the integral constraints (Wiin-Nielsen [28]) can be written as follows:

$$\frac{\partial \zeta}{\partial t}+\mathbf{v}\cdot\nabla\eta=f_0\frac{\partial\omega}{\partial p}+\mathbf{k}\cdot\nabla\times\mathbf{F}, \tag{1}$$

where $\mathbf{v}=\mathbf{k}\times\nabla\psi$ is the non-divergent velocity vector, ψ is the stream function,

$$\zeta=\frac{1}{a^2\cos^2\phi}\frac{\partial^2\psi}{\partial\lambda^2}+\frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\psi}{\partial\phi}\right)=\nabla_s^2\psi,$$

a is the radius of the earth, p is the pressure, ϕ is the latitude, λ is the longitude, $\eta=\zeta+f$, f is the Coriolis parameter, f_0 is the value of f at 45° N., $\omega=(dp/dt)$, and $\mathbf{F}$ is the viscous force per unit mass.

In (1), vertical advection of absolute vorticity and the twisting term are neglected, the absolute vorticity is advected by non-divergent winds, and $\eta(\partial\omega/\partial p)$ is replaced by $f_0(\partial\omega/\partial p)$. If we assume the relation $f_0\psi=\Phi$ where Φ is the geopotential, it is possible (Phillips [19]) to write the thermodynamic equation in the form

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial p}\right)+\mathbf{v}\cdot\nabla\left(\frac{\partial\psi}{\partial p}\right)+\frac{\bar{\sigma}\omega}{f_0}=-\frac{R}{c_p p f_0}H. \tag{2}$$

Where $\sigma=-\alpha(\partial\ln\theta/\partial p)$ is a measure of static stability, the bar over σ indicates the average over a (λ,ϕ) surface, α is the specific volume, θ is the potential temperature, R is the gas constant for dry air, c_p is the specific heat at constant pressure, and H is the rate of change with time of diabatic heating per unit mass.

We shall operate by $-(\partial/\partial p)$ on (1) and by ∇_s^2 on (2), and add to obtain a diagnostic equation for ω,

$$\bar{\sigma}\nabla_s^2\omega+f_0^2\frac{\partial^2\omega}{\partial p^2}=f_0\frac{\partial}{\partial p}(\mathbf{v}\cdot\nabla\eta)-f_0\frac{\partial}{\partial p}(\mathbf{k}\cdot\nabla\times\mathbf{F})$$
$$-f_0\nabla_s^2\left[\mathbf{v}\cdot\nabla\left(\frac{\partial\psi}{\partial p}\right)\right]-\frac{R}{c_p p}\nabla_s^2 H. \tag{3}$$

The axially-symmetric flow in the atmosphere is greatly influenced by the presence of large-scale eddy motion. We shall define this flow as a zonal average of a physical quantity x,

$$x_z(\phi,p,t)=\frac{1}{2\pi}\int_0^{2\pi}x(\lambda,\phi,p,t)d\lambda, \tag{4}$$

so as to obtain an integrated effect of the eddy motion. Thus, performing the zonal averaging operation to (3), we obtain

$$\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left(\frac{\cos\phi\,\partial\omega_z}{\partial\phi}\right)+\frac{f_0^2a^2}{\bar{\sigma}}\frac{\partial^2\omega_z}{\partial p^2}=$$
$$-\frac{f_0}{\bar{\sigma}}\frac{\partial}{\partial p}\left\{\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left[\frac{1}{\cos\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial\phi}\right]\right\}$$
$$+\frac{f_0a}{\bar{\sigma}\cos\phi}\frac{\partial}{\partial p}\left(\frac{\partial F_{\lambda,z}\cos\phi}{\partial\phi}\right)-\frac{g}{(\Delta p)^2}\frac{R\ln\left(\frac{p_2}{p_1}\right)}{2\pi a^2c_p\bar{\sigma}}$$
$$\times\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left\{\cos\phi\frac{\partial}{\partial\phi}\left[\frac{1}{\cos\phi}\frac{\partial TH_{\Delta p}(\phi)}{\partial\phi}\right]\right\}$$
$$-\frac{R}{\bar{\sigma}c_p p}\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial H_z}{\partial\phi}\right), \tag{5}$$

where

$$TH_{\Delta p}(\phi)=\frac{\Delta p c_p a\cos\phi}{g}\int_0^{2\pi}\tilde{T}\tilde{v}\,d\lambda$$

is the horizontal eddy transfer of sensible heat in a layer of thickness Δp; p_1 and p_2 are the pressures at the bottom and top of this layer. $\tilde{T}$ and $\tilde{v}$ are, respectively, the temperature and the meridional component of the wind vector, representative for the layer and g is the acceleration of gravity. The forcing function in (5) consists of four terms. The first two terms are the vertical variation of eddy transfer of zonal momentum and viscous forces to account for the balance of angular momentum in the atmosphere. The third and the fourth terms are the horizontal variations of eddy transfer of sensible heat and diabatic heating, respectively, to account for the balance of the heat budget of the atmosphere.

Further it may be noted here that the eddy transfer of zonal momentum is proportional to the degree of eastward tilt of the trough in the lower latitudes and westward tilt in the higher latitudes on the isobaric surfaces (Starr [23]). Similarly, the eddy transfer of sensible heat is proportional to the degree of westward tilt of the trough in the vertical. The meridional circulation is thus, to some extent, produced by the eddy processes in the atmosphere.

The differential equation for the vertical motion is usually solved using the simplified boundary conditions that the vertical motion is zero at the top and bottom of the atmosphere. We shall consider the vertical motion at the lower boundary due to the roughness of the earth's surface and lifting on the slopes of the mountains. In the frictional layer of the atmosphere, the vorticity equation for the balanced frictional flow is written as:

$$\frac{\partial\omega}{\partial p}=\frac{1}{f_0}\frac{g}{a\cos\phi}\frac{\partial}{\partial p}\left(\frac{\partial\tau_\phi}{\partial\lambda}-\frac{\partial\tau_\lambda\cos\phi}{\partial\phi}\right), \tag{6}$$

where τ_ϕ and τ_λ are the shearing stresses in ϕ and λ directions. Integration of (6) from bottom to the top of the frictional layer on the assumption that the shearing stresses vanish at the top of the frictional layer, gives

$$\omega_m-\omega_f=\frac{g}{f_0 a \cos\phi}\left(\frac{\partial\tau_{\phi,s}}{\partial\lambda}-\frac{\partial\tau_{\lambda,s}\cos\phi}{\partial\phi}\right). \tag{7}$$

Here ω_m and ω_f are the vertical velocities at the bottom and top of the frictional layer, respectively. $\tau_{\phi,s}$ and $\tau_{\lambda,s}$ are the shearing stresses at the lower boundary of the frictional layer.

The magnitude of the surface shear stresses can be estimated by different methods. Mintz [13] computed the mean zonal average of the surface shear stresses by integrating the steady state first equation of motion in the vertical. Phillips [18] evaluated the stress using a semi-empirical formula of the form

$$\boldsymbol{\tau}=C\rho|\mathbf{v}|\mathbf{v}_s, \tag{8}$$

where $C\approx 0.003$ is a non-dimensional constant, the drag coefficient, ρ is the density at the surface of the earth, $|\mathbf{v}|=10$ m. sec.$^{-1}$ and $\mathbf{v}_s$ is the geostrophic velocity at the anemometer level. Substituting (8) in (7), we can write

$$\omega_m-\omega_f=\kappa\zeta_s, \tag{9}$$

where $\kappa=3$ cb.

The zonal average of (9) is

$$\omega_{mz}-\omega_{fz}=-\frac{\kappa}{a\cos\phi}\frac{\partial}{\partial\phi}(u_{s,z}\cos\phi), \tag{10}$$

where ω_{mz} is the vertical velocity at the bottom of the atmosphere due to the sloping terrain. If p_m is the surface pressure of the underlying topography of the earth, the vertical velocity due to the forced lifting is given by the formula

$$\omega_m=\mathbf{v}_s\cdot\nabla p_m, \tag{11}$$

or

$$\omega_{mz}=\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}[\cos\phi(p_m v_s)_z]. \tag{12}$$

Finally, the vertical velocity at the top of the frictional layer is

$$\omega_{fz}=\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}[\cos\phi\,(p_m v_s)_z]+\frac{\kappa}{a\cos\phi}\frac{\partial}{\partial\phi}(u_{s,z}\cos\phi). \tag{13}$$

The time dependent and the time averaged axially-symmetric distribution of the vertical can be computed knowing the forcing function and the boundary conditions.

2.2 THE FORCING FUNCTIONS

It is evident from (5) that meridional circulations are secondary processes in the atmosphere forced by the large-scale eddy processes, diabatic heating, and the viscous forces. The estimation of such secondary processes depends entirely on the accuracy with which the forcing functions are measured or evaluated. The direct or indirect methods of evaluating diabatic heating and viscous forces may not meet the required accuracy for our purpose. We shall, therefore, assume that the flow is adiabatic and frictionless. This will then imply that the fluid is inviscid and hence there is no skin friction at the lower boundary. The only vertical velocity at the lower boundary is due to the irregular terrain over the surface of the earth. It will be seen later that some of the computations of vertical velocity were done assuming that the earth's surface is smooth, so that vertical velocity at the lower boundary is zero.

The first and the third term in the forcing function are functions of eddy transfer of zonal momentum and eddy transfer of sensible heat, respectively. These data were obtained from the calculations made by Wiin-Nielsen, Brown, and Drake [33] and [34]. The data for eddy transfer of zonal momentum were available for two observations per day as a function of latitude from 20°N. to 87.5°N. at the interval of 2.5° latitude at the isobaric levels 850, 700, 500, 300, and 200 mb. for the months of January, April, July, October for 1962 and January 1963. Also the data for January 1964 were available with the three additional levels 1000, 150, and 100 mb. The data for eddy transfer of sensible heat were available for the same grid and observations for the corresponding layers 1000–850, 850–700, and so on, as shown in figure 1. Further, these data were given as a function of wave number including wave numbers 1 to 15.

In addition to the eddy transfer of zonal momentum and sensible heat, one would need the distribution of the static stability σ, in the atmosphere so as to compute the forcing function. Following Wiin-Nielsen [29], $\bar{\sigma}$ was taken as $\bar{\sigma}(p)=(\sigma_0/p^2)$ where $\sigma_0=0.625$ for the troposphere. Or one could use the average values of static stability computed by Gates [5]. It may be mentioned here that the formula suggested by Wiin-Nielsen is closely in agreement with the computed values of the static stability except near 850 mb. and 200 mb.

2.3 METHOD OF SOLUTION

The following transformation of the independent variables was made to write (5) in the convenient form:

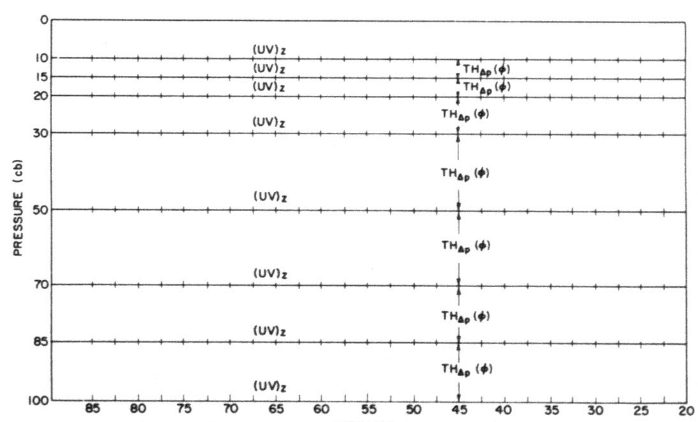

FIGURE 1.—Schematic diagram showing the data grid for which the eddy transfer of zonal momentum and the eddy transfer of sensible heat were available for the month of January 1964.

$$\sin\phi=\mu$$

and

$$p_*=\frac{p}{p_0},$$

where

$p_0=100$ cb., the surface pressure

so that

$$0\leq\mu\leq 1 \text{ for } 0\leq\phi\leq\frac{\pi}{2}$$

and

$$0\leq p_*\leq 1 \text{ for } 0\leq p\leq 100 \text{ cb}.$$

Equation (5) for adiabatic and frictionless flow then takes the form:

$$\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\omega_z}{\partial\mu}\right]+q^2\frac{\partial^2\omega_z}{\partial p_*^2}=M(\mu, p_*) \tag{14}$$

where

$$M(\mu, p_*)=-\frac{f_0}{\bar{\sigma}p_0}\frac{\partial}{\partial p_*}\left\{\frac{\partial^2}{\partial\mu^2}[(1-\mu^2)(uv)_z]\right\}$$

$$-\frac{gR}{2\pi a^2}\frac{\ln\left(\frac{p_2}{p_1}\right)}{(\Delta p)^2 c_p\bar{\sigma}}\frac{\partial}{\partial\mu}\left\{(1-\mu^2)\frac{\partial^2}{\partial\mu^2}[TH_{\Delta p}(\phi)]\right\}$$

and

$$q^2=\frac{f_0^2a^2}{\bar{\sigma}p_0^2}.$$

Equation (14) can be solved by relaxation methods, but it is convenient to solve it by separation of variables to reduce the truncation errors involved in finite differencing.

Let $\omega_z(\mu, p_*)$ be expressed as a finite series of Legendre polynomials;

$$\omega_z(\mu, p_*)=\sum_{n=0}^{N}A_n(p_*)P_n(\mu). \tag{15}$$

Similarly, the forcing function can be expressed as

$$M(\mu, p_*)=\sum_{n=0}^{N}B_n(p_*)P_n(\mu). \tag{16}$$

$P_n(\mu)$ are orthogonal functions over $-1\leq\mu<1$, that is,

$$\int_{-1}^{1}P_n(\mu)P_m(\mu)d\mu=\begin{cases}\frac{2n+1}{2} & \text{for } n=m\\ 0 & \text{for } n\neq m.\end{cases} \tag{17}$$

Hence $B_n(p_*)$ can be computed knowing $M(\mu, p_*)$ over $-1\leq\mu\leq 1$ using the orthogonal property (17) from the relation

$$B_n(p_*)=\frac{2n+1}{2}\int_{-1}^{1}M(\mu, p_*)P_n(\mu)d\mu. \tag{18}$$

It was mentioned earlier in section 2.2 that the data to compute $M(\mu, p_*)$ were available only from 20°N. to 87.5°N. The data for eddy transfers were extrapolated from 20°N. to the Equator by a biquadratic even polynomial in ϕ such that there is no flux across the Equator and the function and its derivatives are continuous at 20°N. The value at the Pole was obtained by Newton's interpolation formula knowing the value at 87.5° N. which is the same on either side of the Pole. Further, it was assumed that the eddy transfers are symmetric around the Equator, so

$$M(\mu, p_*)=M(-\mu, p_*) \qquad 0\leq\mu\leq 1. \tag{19}$$

Hence, (18) can then be written as

$$B_n(p_*)=(2n+1)\int_0^1 M(\mu, p_*)P_n(\mu)d\mu. \tag{20}$$

Here n has to be an even integer to satisfy (20) because $P_n(\mu)=(-1)^nP_n(-\mu)$. The Legendre polynomials $P_n(\mu)$ were computed from the recurrence relation

$$P_n(\mu)=\frac{2n-1}{n}\mu P_{n-1}(\mu)-\frac{n-1}{n}P_{n-2}(\mu) \text{ for } n\geq 2$$

and by definition,

$$P_n(\mu)=\frac{1}{2^n}\frac{1}{n!}\frac{d^n}{d\mu^n}(\mu^2-1)^n, \tag{21}$$

we have

$$P_0(\mu)=1;\ P_1(\mu)=\mu.$$

The accuracy of computing the Legendre polynomials was checked by their orthogonality property (17). The error never exceeded 10^{-4}, when integration was evaluated by Simpson's quadrature formula.

The time averaged eddy transports over the months are smooth functions of ϕ and p. The eddy transfer of zonal momentum is differentiated twice with respect to μ and once with respect to p_*, and the eddy transfer of sensible heat is differentiated three times with respect to μ in (14) to get the forcing functions. The differentiation was done numerically, using centered finite differencing. The resulting forcing function was not a smooth function. It was, therefore, necessary to smooth $M(\mu, p_*)$ such that its main features are retained and the order of magnitude is not affected. This was achieved by taking a smaller number of polynomials than required to represent it completely. $N=10$ was enough to represent the main features of the forcing function without affecting its order of magnitude. The integration in (20) was evaluated by Simpson's quadrature formula.

Now, (14) can be written for a particular Legendr) polynomial after substituting for $\omega_z(\mu, p_*)$ and $M(\mu, p_*$e from (15) and (16), respectively, as follows:

$$A_n(p_*)\frac{d}{d\mu}\left[(1-\mu^2)\frac{dP_n(\mu)}{d\mu}\right]$$
$$+q^2\frac{d^2A_n(p_*)}{dp_*^2}P_n(\mu)=B_n(p_*)P_n(\mu). \tag{22}$$

$P_n(\mu)$ satisfies the Legendre equation

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{dP_n(\mu)}{d\mu}\right]+n(n+1)P_n(\mu)=0. \tag{23}$$

Thus, in view of (23), we may write (22) in the form

$$q^2 \frac{d^2A_n(p_*)}{dp_*^2} - n(n+1)A_n(p_*) = B_n(p_*). \tag{24}$$

Equation (24) was solved by ordinary centered finite difference method with boundary conditions that $A_n(0)=0$ and $A_n(1)=0$ (i.e., $\omega_z=0$ at the top and bottom of the atmosphere), for the months of January, April, July, October 1962 and January 1963. For January 1964, however, $A_n(1)$ was determined from the vertical velocity due to the terrain of the earth's surface. In (12), the vertical velocity produced by the orography depends on the surface pressure and meridional component of wind. The surface pressure was computed from the hydrostatic equation with the height of the topography obtained from the data compiled by Berkovsky and Bertoni [1]. For January 1964, the data were available to compute the surface geostrophic meridional component of the wind. $\omega_z(\mu, p_*)$ was then computed from (15). One should note here that $\omega_z(\mu, p_*)$ has to satisfy the physical constraint imposed by the equation of continuity. The area average of ω over any isobaric surface which does not intersect the ground is zero.

That is

$$\bar{\omega} = \frac{1}{2\pi} \int_0^{\pi/2} \int_0^{2\pi} \omega \cos\phi \, d\phi \, d\lambda$$

$$= \int_0^{\pi/2} \omega_z \cos\phi \, d\phi = \bar{\omega}_z = 0. \tag{25}$$

But

$$\bar{\omega}_z = \sum_{n=0}^{N} A_{2n}(p_*) \int_0^1 P_{2n}(\mu) d\mu$$

$$= A_0(p_*) \int_0^1 P_0(\mu) d\mu + \sum_{n=1}^{N} A_{2n}(p_*) \int_0^1 P_{2n}(\mu) d\mu, \tag{26}$$

$$\int_0^1 P_0(\mu) d\mu = 1, \tag{27}$$

and

$$\int_0^1 P_{2n}(\mu) d\mu = \frac{1}{4n+1} \int_0^1 \left[\frac{dP_{2n+1}(\mu)}{d\mu} - \frac{dP_{2n-1}(\mu)}{d\mu} \right] d\mu$$

$$= \frac{1}{4n+1} [P_{2n+1}(1) - P_{2n+1}(0)$$

$$- P_{2n-1}(1) + P_{2n-1}(0)] = 0, \tag{28}$$

since

$$P_{2n+1}(0) = P_{2n-1}(0) = 0$$

and

$$P_{2n+1}(1) = P_{2n-1}(1) = 1.$$

Hence, $A_0=0$ to satisfy (25). Therefore (15) may be rewritten as

$$\omega_z(\mu, p_*) = A_2(p_*)P_2(\mu) + \ldots + A_n(p_*)P_n(\mu). \tag{29}$$

The zonally averaged vertical velocity distribution was computed from (29) from the Equator to the North Pole at intervals of 2.5° and for the levels 775, 600, 500, and 250 mb. for the 4 months of 1962 representing the four seasons and January 1963, whereas vertical resolution was better in January 1964, and the vertical velocity was computable for levels 925, 775, 600, 400, 250, 175, and 125 mb. from (29) and at 1000 mb. due to the orographic lifting.

The axially-symmetric flow consists of a pure zonal flow u_z and a meridional flow with velocity $\mathbf{v}_m = v_z\mathbf{j} + \omega_z\mathbf{k}$ which is identical in all meridional planes. Once the ω_z is computed by the method described above, the zonally averaged meridional component can be computed from the equation of continuity.

The zonally averaged equation of continuity in spherical coordinates is

$$\frac{1}{a \cos\phi} \frac{\partial}{\partial\phi} (v_z \cos\phi) + \frac{\partial\omega_z}{\partial p} = 0. \tag{30}$$

Integrating (30) from the North Pole to any latitude ϕ_i we get

$$\int_{\pi/2}^{\phi_i} \frac{\partial}{\partial\phi} (v_z \cos\phi) d\phi = -a \int_{\pi/2}^{\phi_i} \frac{\partial\omega_z}{\partial p} \cos\phi \, d\phi. \tag{31}$$

Since $v_z\cos\phi=0$ at the North Pole (i.e., $\phi=\pi/2$),

$$v_z(\phi_i, p) = -\frac{a}{\cos\phi_i} \int_{\pi/2}^{\phi_i} \frac{\partial\omega_z}{\partial p} \cos\phi \, d\phi. \tag{32}$$

The term $(\partial\omega_z/\partial p)$ was computed by centered finite differencing and the integration was carried out using the trapezoidal rule. The zonally-averaged meridional component of velocity was obtained from the Equator to the North Pole at intervals of 2.5° and for levels 887.5, 687.5, 500, 325, and 125 mb. in 1962 and January 1963. For January 1964, it was computed for the levels 962.5, 850, 687.5, 500, 325, 212.5, 150, and 62.5 mb.

3. PRESENTATION AND DISCUSSION OF RESULTS

It was mentioned earlier that the data were available only from 20°N. to 87.5°N. It was necessary to extrapolate the data from 20°N. to the Equator to be able to employ a convenient method of solution. One cannot be certain of the results in the extrapolated region, so the results for the region 20°N. to 87.5°N. will be presented here.

3.1 MEAN MERIDIONAL CIRCULATIONS IN AN ADIABATIC FRICTIONLESS ATMOSPHERE

The axially-symmetric vertical velocity distribution, ω_z, as a function of latitude and pressure for the month of January 1963 is shown in figure 2. To avoid a further assumption about the density variations in the atmosphere the results are given in the units 10^{-5} mb. sec.$^{-1}$ Hence the positive values indicate a descending motion while the negative values represent an ascending motion. The maximum values of the ascending or the descending motion are found at the level of non-divergence in the atmosphere. Intense downward flux of mass is found around 35°N., the region of the subtropical high pressure belts, while the upward flux of mass occurs in the region

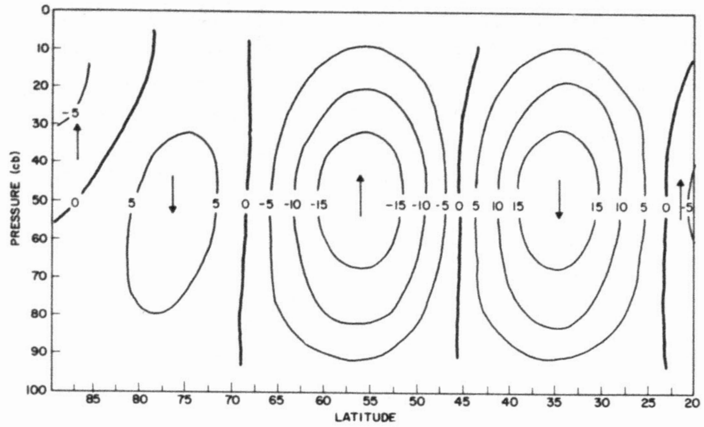

FIGURE 2.—The zonally averaged vertical velocity, ω_z, for the month of January 1963 as a function of latitude and pressure in the unit: 10^{-5} mb. sec.$^{-1}$

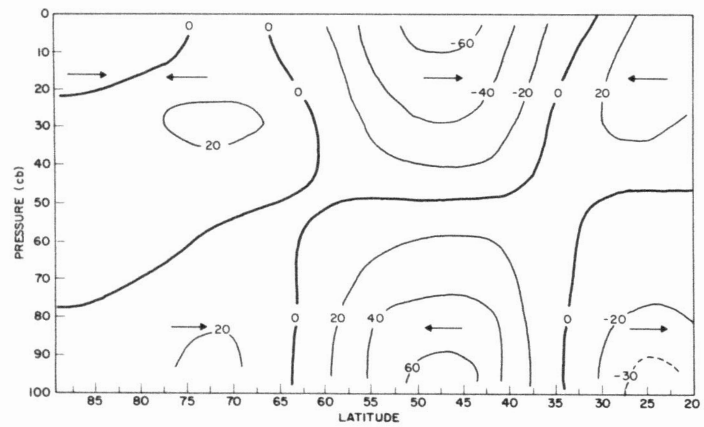

FIGURE 3.—The zonally averaged meridional velocity, v_z, for the month of January 1963 as a function of latitude and pressure in the unit: cm. sec.$^{-1}$

of the low pressure belts near 55°N. The lines of zero vertical motion are the centers of the meridional circulation cells. Thus the tropical direct cell is situated around 22.5°N., the reverse middle latitude cell near 45°N., and the polar direct cell around 70°N. The intensity of the cells decreases from the Equator toward the Pole. The rising vertical motion found in the polar region near 400 mb. is an indication of opposite meridional circulations in the stratosphere. Our result is thus in agreement with a stratospheric circulation postulated by Kuo [10]. Since our data are restricted to levels below the 200-mb. level such reverse circulations are not seen in the middle latitudes and the subtropical regions.

The distribution of the meridional component of the axially-symmetrical flow for the month of January 1963 is illustrated as a function of latitude and pressure in figure 3. The meridional velocities are given in the units cm. sec.$^{-1}$ The negative and the positive values indicate southward and northward flow, respectively. In the subtropical region the equatorward flow extends up to 35°N. in the lower troposphere while the poleward flow at higher levels extending up to the same latitude describes a part of the tropical direct cell. The reverse cell in the middle latitudes has a maximum northward flow of 60 cm. sec.$^{-1}$ around 47°N. in the lowest layer and the maximum reverse flow of the same intensity near the top of the troposphere. The polar direct cell being much weaker than the middle latitude cell has a maximum equatorward flow of 20 cm. sec.$^{-1}$ near the ground and a maximum poleward flow of the same intensity near 300 mb. The equatorward flow near 100 mb. in the polar region is again an indication of the reverse cell in the stratosphere. These computations of the mean meridional circulations are in agreement with the results obtained by Mintz and Lang [14], and Holopainen [7], except between 25–20°N.

3.2 SEPARATE EFFECTS OF EDDY TRANSFERS OF ZONAL MOMENTUM AND SENSIBLE HEAT ON THE MEAN MERIDIONAL CIRCULATIONS

We have seen from (14) that the forcing function $M(\mu,p_*)$ is the sum of two terms. The first term is a function of eddy transfer of zonal momentum and the second is a function of eddy transfer of sensible heat. Henceforth we shall refer to the former as $f[(uv)_z]$ and the latter as $f[(Tv)_z]$. The role of one of the terms in the mean meridional circulations can be determined by letting the other be identically zero in the (μ,p_*) plane.

The results of such calculations show that the influence of $f[(uv)_z]$ and $f[(Tv)_z]$ on the circulation pattern is similar to their joint effect (i.e., similar to figs. 2 and 3). In order to compare the intensities of the circulations produced by $f[(uv)_z]$ and $f[(Tv)_z]$ we have computed the mass circulation, $M^*(\phi)$, across any latitude ϕ by a formula

$$M^*(\phi)=\frac{2\pi a \cos\phi}{g}\int_{p_0}^{p} v_z dp \tag{33}$$

where p is the pressure at which the mass circulation reverses its direction in the troposphere.

Figure 4 gives the mass circulation as a function of latitude in units 10^6 tons sec.$^{-1}$ forced by the two terms separately and collectively. The positive mass circulation in the middle latitudes indicates the northward flow in the lower troposphere and the equal amount of southward flow in the upper troposphere, whereas the opposite is true for the negative circulation in the lower and higher latitudes. The dashed curve shows the mass circulation due to $f[(uv)_z]$ while the dash-dotted curves gives that due to $f[(Tv)_z]$. Their combined effect is shown by a continuous curve. The proportion of the mass circulation due to $f[(uv)_z]$ or $f[(Tv)_z]$ to the total is a function of latitude. Comparing the absolute values of the mass circulation $f[(uv)_z]$, by and large, explains ⅔ of the total mass circulation while only ⅓ of it is explained by $f[(Tv)_z]$.

3.3 MEAN MERIDIONAL CIRCULATIONS IN WAVE NUMBER REGIME

It was mentioned earlier that the data for computing $f[(uv)_z]$ and $f[(Tv)_z]$ were available as a function of wave

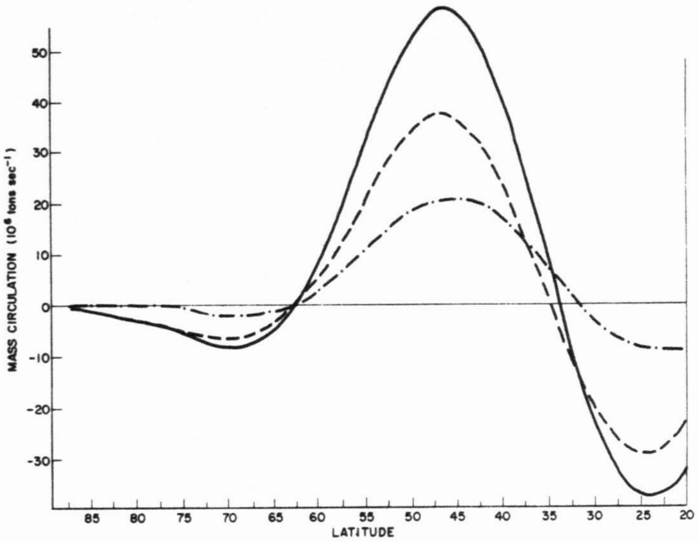

FIGURE 4.—The mass circulation in the lower troposphere for the month of January 1963 as a function of latitude in the unit: 10^6 tons sec.$^{-1}$ Solid curve: the mass circulation produced by both $f((uv)_z)$ and $f((Tv)_z)$. Dashed curve: the mass circulation produced by $f((uv)_z)$. Dash-dotted curve: the mass circulation produced by $f((Tv)_z)$.

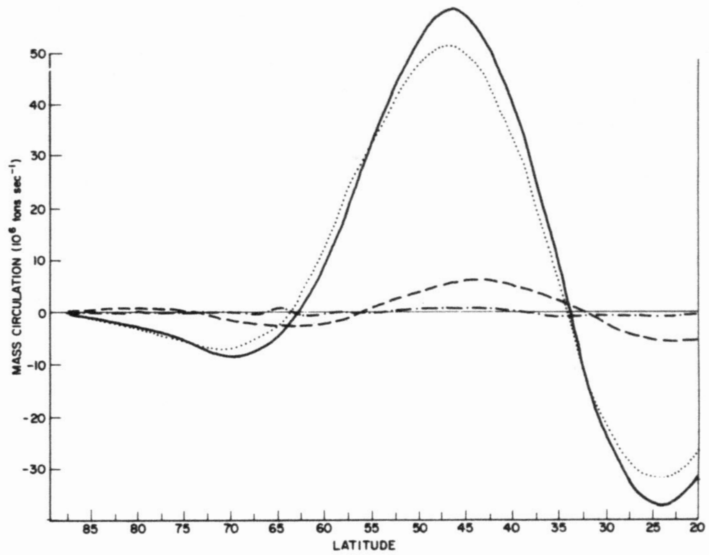

FIGURE 5.—The mass circulation in the lower troposphere for the month of January 1963. Solid curve: the mass circulation produced by all scales of motion. Dotted curve: the mass circulation produced by the planetary waves. Dashed curve: the mass circulation produced by the medium waves. Dash-dotted curve: the mass circulation produced by the short waves. Arrangement and units as in figure 4.

number including wave number 1 to 15. It is however cumbersome to present the effect of all 15 waves. We have, therefore, grouped the 15 components in three groups. The first, consisting of wave numbers 1 to 4, represents the planetary or long waves. The next group, consisting of wave numbers 5 to 8, is usually called medium waves, while the final group, consisting of wave numbers 9 to 15, represents the short waves. The circulation patterns produced by different scales of motion are strikingly similar to their total effect. The similar circulation pattern for different scales of motion is not surprising, because eddy transports for both momentum and heat have similar distributions for all the three wave groups (Wiin-Nielsen, Brown, and Drake [34]). The difference lies in the intensity of the circulation for different scales. We shall now refer to figure 5 to compare the intensities of the circulations produced by different scales of motion. Here the dotted curve gives the distribution of the mass circulation in the lower troposphere as a function of latitude due to the long waves, dashed curve for the medium waves, and the dash-dotted curve for the short waves. The total effect is shown by the continuous curve. Comparing the absolute values of the mass circulation over the entire region, 20°N. to 87.5°N., it is found that the long waves account for 86 percent of the total mass circulation, while the medium waves explain 12 percent and the remaining 2 percent are due to the short waves.

3.4 SEASONAL VARIATIONS OF MEAN MERIDIONAL CIRCULATIONS

The investigation was carried out for 4 months, January, April, July, and October of 1962, to represent the four seasons of the year. These computations were arranged in the same manner as for January 1963, to obtain the separate effects of $f[(uv)_z]$ and $f[(Tv)_z]$, and also to find meridional circulations in the wave number regime. It is not necessary to present all these results, but certain interesting points may be summarized as follows:

(1) The conclusions drawn in section 3.2 about the relative importance of $f[(uv)_z]$ and $f[(Tv)_z]$ for the month of January 1963 hold good irrespective of the season in the year 1962.

(2) In January 1962 the ratio of the contribution by long, medium, and short wave groups, to the total circulation was 0.55, 0.31, and 0.14, respectively, as compared to 0.86, 0.12, 0.02 in the month of January 1963. The extreme dominance of the very long waves was characteristic of January 1963 (Wiin-Nielsen, Brown, and Drake [34]).

(3) For April, July, and October 1962 the meridional circulations for different scales of motion had, by and large, the same proportion but different from that of January 1962 or 1963. Here the long waves explained only 32 percent of the total circulation, whereas the medium waves explained 57 percent and the remaining 11 percent accounted for by the short waves.

The medium waves represent in general the baroclinically unstable waves in the atmosphere. It is satisfactorily established by empirical studies (Wiin-Nielsen, Brown, and Drake [33] and [34]) that these scales dominate the eddy transfer processes in the spring, summer, and fall seasons. The above results are thus in agreement with earlier studies.

For the discussion of the seasonal variation of the mean meridional circulation we shall consider the circulation forced by the total forcing function and summed over all scales of motion.

Figures 6, 8, 10, and 12 show the distribution of the vertical velocity, as a function of latitude and pressure, for the months of January, April, July, and October 1962, respectively. Similarly, figures 7, 9, 11, and 13 illustrate the corresponding distribution of the meridional velocity for the respective months. Since the major features of the meridional circulations are the same for all the representative months of the seasons we shall avoid the repetitious description of these figures and discuss only certain points regarding the seasonal variation of the circulation.

(1) The three-cell mean meridional circulations prevail in all the seasons, except possibly during summer.

(2) There seems to be a trend to move these cells northward from winter to spring and to reach their extreme northern position in the summer when the polar direct cell has almost disappeared. From summer to fall these cells move southwards regaining their extreme southern position in winter, completing an annual cycle of oscillation.

(3) The intensity of the circulation gradually decreases from winter to the spring and attains its minimum in summer. The intensity starts to increase from summer to fall and reaches its maximum in winter. The mass circulations across the latitudes for January, April, July, and October 1962 are shown in figure 14. Comparing the absolute values of the mass circulations we find that the circulations in January are three times as intense as those in July. Further, that the intensity of the circulations in April is almost the same as in October, but it is twice as much as that in July.

3.5 THE EFFECT OF THE LOWER BOUNDARY CONDITIONS ON MEAN MERIDIONAL CIRCULATIONS

So far we have discussed the results of mean meridional circulation based on an adiabatic and frictionless model with the simplified boundary conditions, that the vertical

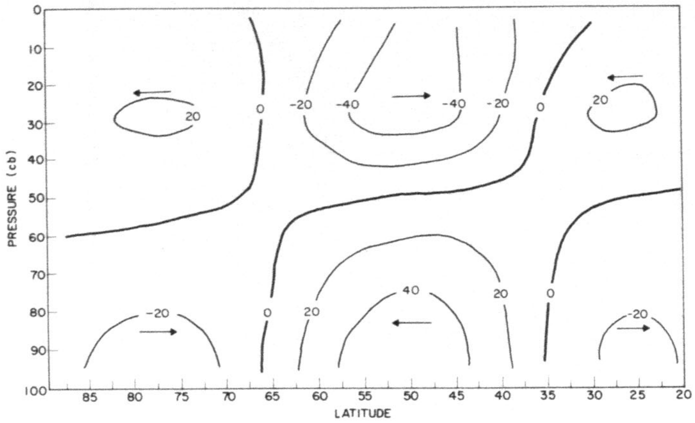

FIGURE 7.—The zonally averaged meridional velocity, v_z, for the month of January 1962. Arrangement and units as in figure 3.

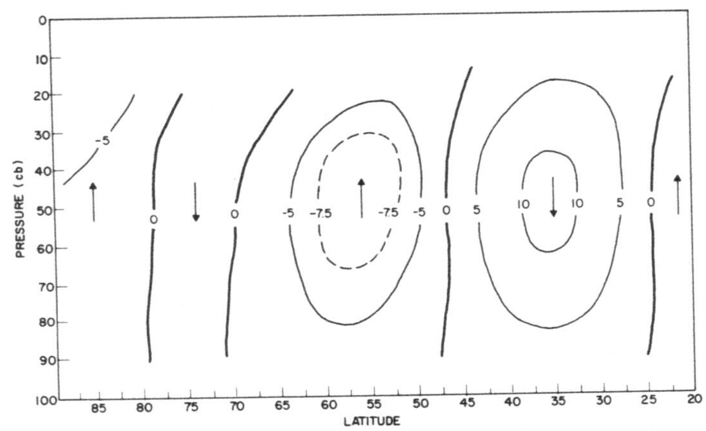

FIGURE 8.—The zonally averaged vertical velocity, ω_z, for the month of April 1962. Arrangement and units as in figure 2.

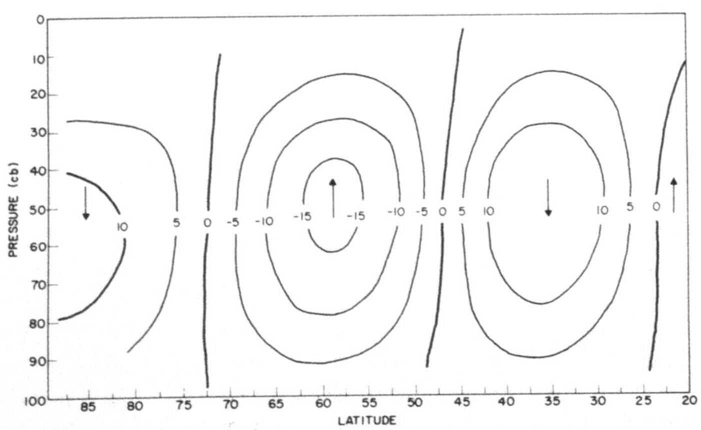

FIGURE 6.—The zonally averaged vertical velocity, ω_z, for the month of January 1962. Arrangement and units as in figure 2.

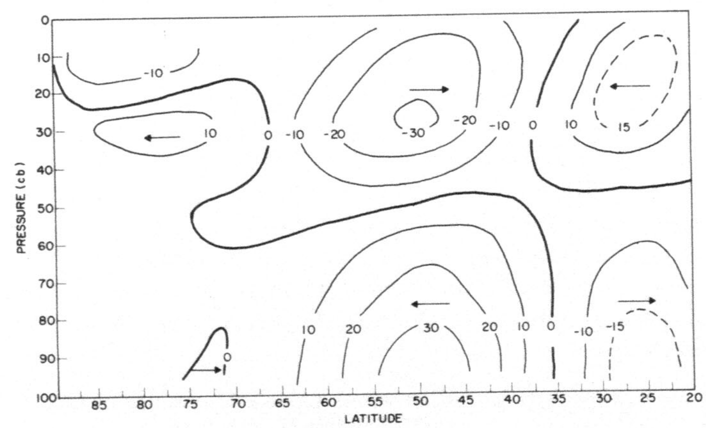

FIGURE 9.—The zonally averaged meridional velocity, v_z, for the month of April 1962. Arrangement and units as in figure 3.

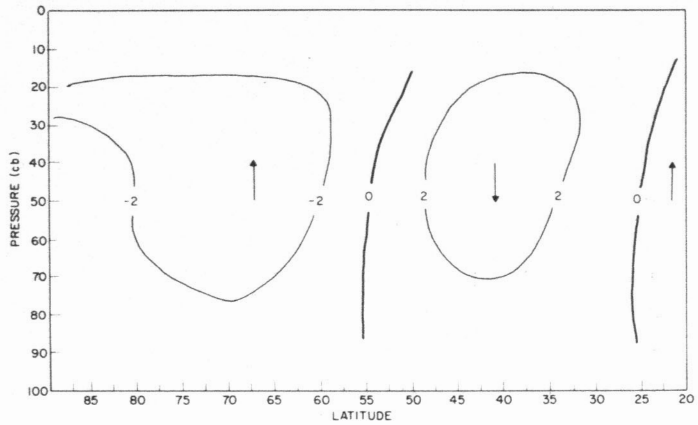

FIGURE 10.—The zonally averaged vertical velocity, ω_z, for the month of July 1962. Arrangement and units as in figure 2.

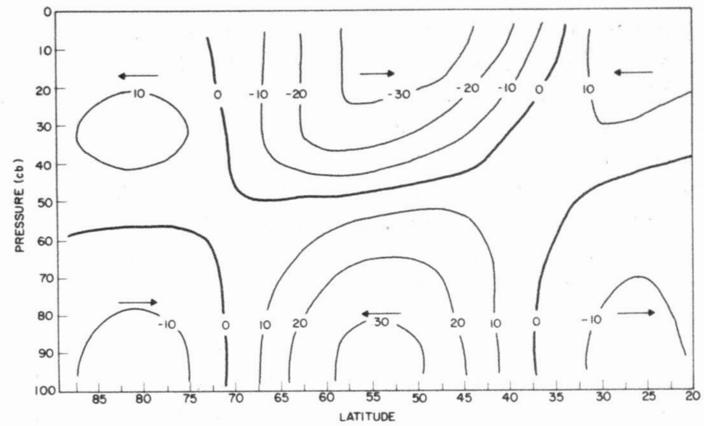

FIGURE 13.—The zonally averaged meridional velocity, v_z, for the month of October 1962. Arrangement and units as in figure 3.

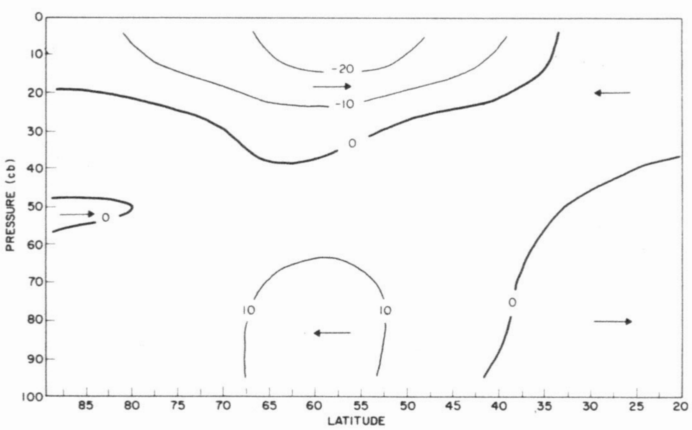

FIGURE 11.—The zonally averaged meridional velocity, v_z, for the month of July 1962. Arrangement and units as in figure 3.

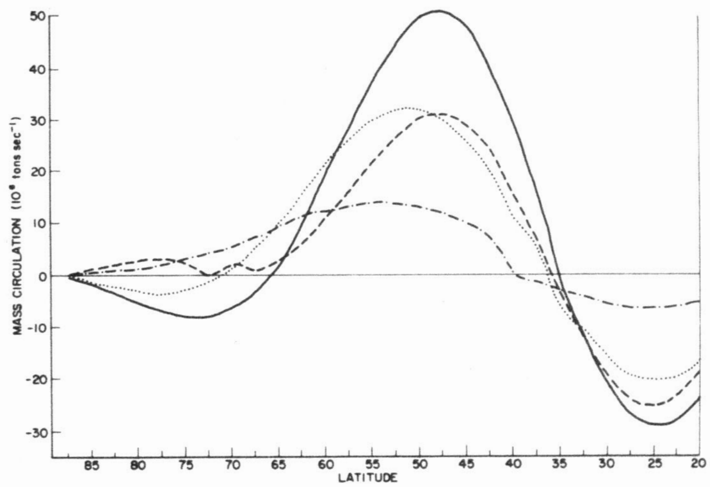

FIGURE 14.—Solid curve: the mass circulation for the month of January 1962. Dashed curve: the mass circulation for the month of April 1962. Dash-dotted curve: the mass circulation for the month of July 1962. Dashed curve: the mass circulation for the month of October 1962. Arrangement and units as in figure 4.

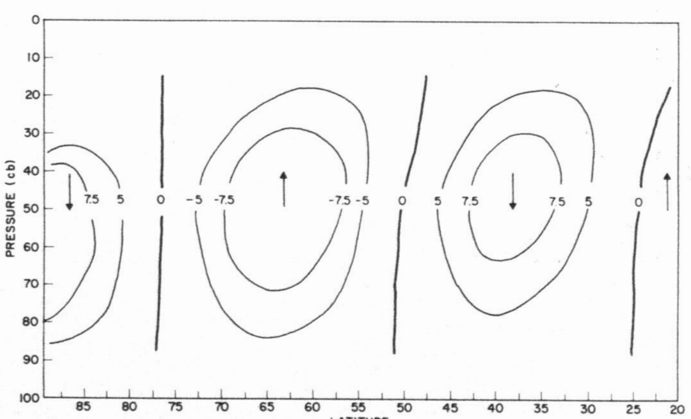

FIGURE 12.—The zonally averaged vertical velocity, ω_z, for the month of October 1962. Arrangement and units as in figure 2.

velocity, ω_z, is zero at the lower and at the upper boundary of the atmosphere. It is clear from section 2.1 that a realistic model should include the diabatic heating, the viscous forces, and the effect of topography and friction at the lower boundary. In addition one should also have a greater vertical resolution in the data since the present method of solution uses finite differencing in the vertical. Although it is not possible to incorporate the diabatic heating and viscous forces in a realistic manner, the effects of topography and friction at the lower boundary can be computed from (13). Further, we shall recall here that a greater vertical resolution in data was available in January 1964. The vertical velocity due to the topography, ω_{mz}, evaluated from (12), was expressed in a series of Legendre polynomials to determine $A_n(1)$. The forcing function $M(\mu, p_*)$ was represented by the first 10 polynominals. It was therefore necessary to express ω_{mz} with the same number of polynomials. This helped to smooth out the errors in ω_{mz} due to the finite differencing.

The vertical velocity, ω_z, as a function of latitude and pressure for January 1964, in units 10^{-5} mb. sec.$^{-1}$, is shown in figure 15. The interesting feature of this figure is that the level of maximum vertical velocity is much closer to the observed level of non-divergence in the atmosphere (600 mb.) as compared to that of January 1962 or 1963. Here the stratospheric circulations in the polar region are clearly seen.

The meridional velocity, v_z, as a function of latitude and pressure for January 1964 in the units cm. sec.$^{-1}$ is shown in figure 16. The tropospheric circulations have a pattern similar to those found in January 1962 or 1963. In the polar stratosphere a complete circulation of an indirect cell is seen. There is also an indication of a stratospheric reverse circulation in the subtropical region.

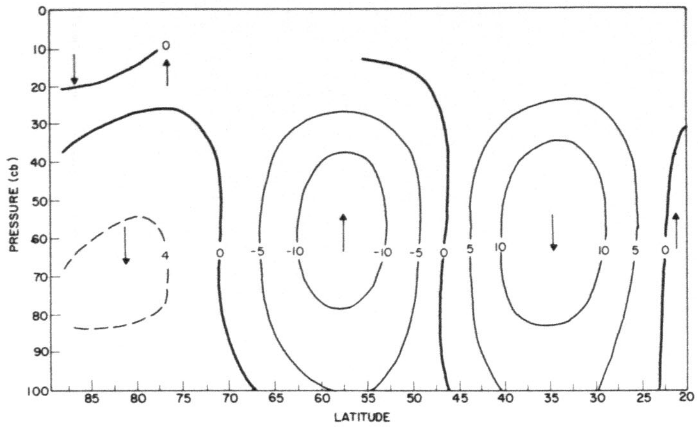

FIGURE 15.—The zonally averaged vertical velocity, ω_z, for the month of January 1964. Arrangement and units as in figure 2.

4. DAILY VARIATIONS OF MERIDIONAL CIRCULATIONS

The atmosphere is constantly in a state of complex motion, changing with time. The periodicities, such as the annual cycle of variations, are well hidden by the transient motion on a day-to-day basis. The periodicities like atmospheric tides and diurnal variation of temperature are hardly detectable in the large-scale upper air flow. There is considerable doubt as to whether any detectable periodicities occur in the tropospheric weather data (Ward and Shapiro [26]). However, there exists an oscillation of the general circulation, especially in the middle and the high latitudes, known as the index cycle (Namias and Clapp [15]). The index cycle refers to the oscillation between strong and weak westerlies in the middle latitudes. High index represents a state of circulation with strong westerlies in the middle latitudes and relatively weak eddies, whereas a low index has relatively weak westerlies but well developed eddies (Smagorinsky [22]). Therefore the eddy transport of momentum and heat must be relatively less intense during the high index period and relatively more intense during the low index period. The meridional circulations in our present model are forced by these eddy transport processes and hence one should expect similar changes in the intensity of the mean meridional circulations. Further the different stages in the index cycle take place gradually and hence one should expect the mean meridional circulations to be a smooth function of time.

To date, the study of variations in the meridional circulations on a daily basis has not been undertaken. This is so because the indirect method of computations is based on the steady-state models. However, Defant and van de Boogaard [4] and van de Boogaard [24] computed meridional circulations by the direct method for one synoptic observation between Equator and 40°N. Their results show good agreement with other investigators' (Palmén, Riehl, and Vuorela [17]) results based on seasonal averages. It seems, therefore, that the meridional

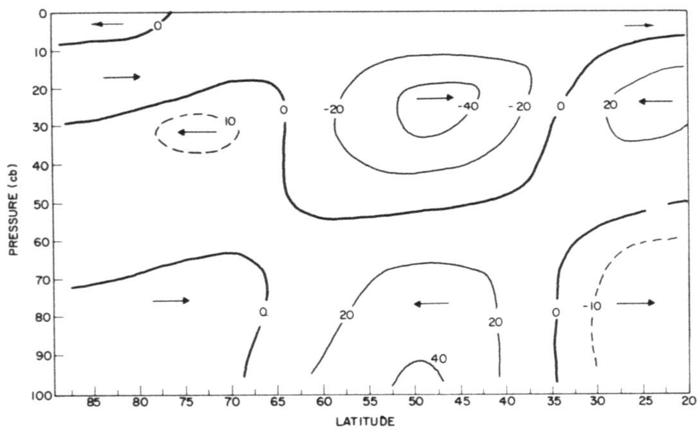

FIGURE 16.—The zonally averaged meridional velocity, v_z, for the month of January 1964. Arrangement and units as in figure 3.

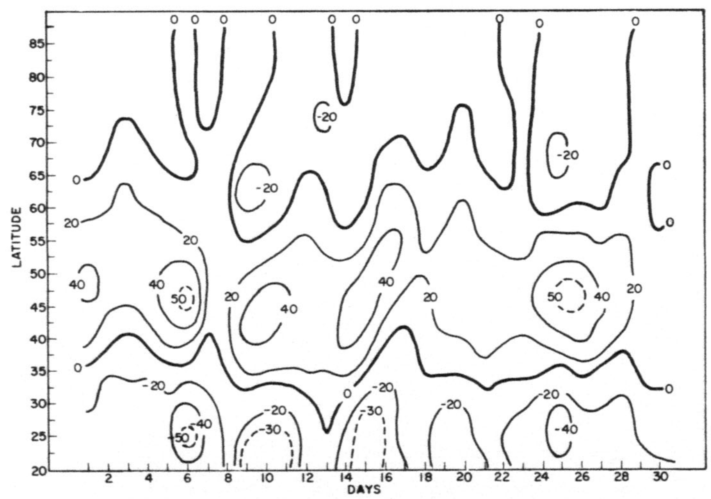

FIGURE 17.—The daily variation of the mass circulation in the lower troposphere as a function latitude and time for the month of January 1964 in the unit: 10^6 tons sec.$^{-1}$

circulations computed on a daily basis may have circulation patterns consistent with the mean monthly circulations discussed earlier.

We shall recall that our method of computing the meridional circulation is based on a diagnostic equation. Hence, it is possible to compute the meridional circulations whenever the forcing function is known.

The eddy transfers of zonal momentum and sensible heat were computed from the height data for two observations per day (00 GMT and 12 GMT) for January 1964. The meridional circulations were computed by the method described in section 2.3 with the boundary conditions that $\omega_z=0$ at the top and bottom of the atmosphere.

To present these results as a function of time we have computed the mass circulation in the lower troposphere from (33). The mass circulations as a function of latitude and time in the units 10^6 tons sec.$^{-1}$ are illustrated in figure 17. The negative values indicate the equatorward mass flow while the positive values indicate the poleward flow. The curve, representing the zero mass circulation in the lower latitudes, shows the northern extent of the tropical direct cell. One might note here that if the tropical cell moves toward the north the intensity of the circulation in middle latitudes decreases. Such a behavior was also noticed in the seasonal variation of the meridional circulation (section 3.4). The curve for zero mass circulation in the higher latitudes shows the northern extent of the middle latitude reverse cell. This curve is very irregular and on certain days the reverse cell extends all the way up to the North Pole. Considering the variation of the intensity of the circulation, we notice that there are five distinct periods of maximum intensity. However, there is no periodicity to their occurrence, but the time series analysis based on a long record of such data might show a substantial peak in the spectrum. The intensities of the eddy transports were checked from the computed data for the period on which the meridional circulation had relative maxima. It was found that there were indeed maximum values during these periods. An independent check can be made on the intensity of the meridional circulation considering the high and low index situations. The meridional circulation should be more intense during the low index situation than in the high index situation. One of the measures of the high and the low index is the magnitude of the maximum zonally averaged west-east component of wind, u_z(max). Figure 18 shows u_z(max) in m. sec.$^{-1}$ as a function of time. Comparing the period of minimum strength of the zonal winds with figure 17 we notice that these periods agree with the periods of maximum intensity of the meridional circulation. Similarly, if we note the periods of maximum zonal winds, we notice that these periods agree with the periods of minimum intensity of the circulations. These results obtained on a daily basis indicate that the present model is sufficiently accurate to give reasonable estimates of the meridional circulations which may be used for further studies of the energetics of the atmosphere.

5. ENERGETICS OF THE ZONAL FLOW

5.1 PRELIMINARY REMARKS

Several investigations have been made in recent years of the energy cycle of the atmosphere. Lorenz [11] introduced the subdivision of both available potential and kinetic energies into their zonal and eddy (deviation from zonal) contributions. Besides the energy levels of these quantities, the conversions between the zonal and eddy available potential energy, $C(A_z, A_e)$, and between the eddy and zonal kinetic energy, $C(K_e, K_z)$, can be computed from routine observed data. Saltzman and Fleisher [20] computed $C(K_e, K_z)$ from height data for 500 mb. Wiin-Nielsen, Brown, and Drake [33, 34] made an extensive study of both $C(A_z, A_e)$ and $C(K_e, K_z)$ for different scales of motion and described the vertical distribution of these quantities. Krueger, Winston, and Haines [9] made a study of $C(A_z, A_e)$ over a 5-yr. period. A comparison between these investigations shows good agreement on the magnitude and direction of these conversions. The conversion between the zonal available potential energy and the zonal kinetic energy, $C(A_z, K_z)$, and the conversion between the eddy available potential energy and the eddy kinetic energy, $C(A_e, K_e)$ depend on the distribution of vertical motions in the atmosphere. Since the vertical velocities are impossible to observe for the large-scale motion, it is necessary to compute them indirectly. The National Meteorological Center obtains the vertical velocities as a by-product of baroclinic numerical prediction model on the routine basis. Wiin-Nielsen [30]; Saltzman and Fleisher [21] and Krueger, Winston, and Haines [9] computed $C(A_z, K_z)$ and $C(A_e, K_e)$ for a layer 850–500 mb. using these vertical velocities at 600 mb. These investigations on $C(A_z, K_z)$ and $C(A_e, K_e)$ agree with each other to some extent. Jensen [8] computed the vertical velocities by a so-called adiabatic method to describe the vertical distribution of $C(A_e, K_e)$. Apparently these results are in error as pointed out by Wiin-Nielsen [31]. Since the results of these calculations depend entirely on the method of computing the vertical velocities, it is desirable to pursue further studies on the conversions. A discussion of the vertical variation of $C(A_z, K_z)$ is presented in section 5.3.

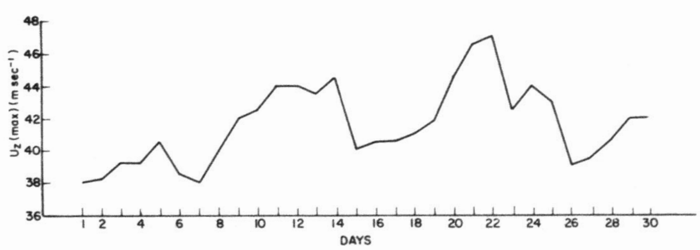

FIGURE 18.—The variation of the daily values of the maximum zonal wind as a function of time for the month of January 1964 in the unit: m. sec.$^{-1}$

The generation of the zonal available potential energy, $G(A_z)$, and the eddy available potential energy, $G(A_e)$ depend on the distribution of diabatic heating in the atmosphere. Wiin-Nielsen and Brown [32] were the first to obtain estimates of $G(A_z)$ and $G(A_e)$ computing them for a layer of 800–600 mb. Brown [2] extended the study to four seasons of the year 1961 with an improvement in the lower boundary condition.

We have made an attempt here to compute a diabatic heating in the meridional plane and hence to obtain a vertical distribution of $G(A_z)$. The results are described in the following section.

5.2 DIABATIC HEATING

The dominant heating factors in the atmosphere are radiation, latent heat due to the phase change of water, and the turbulent exchange of heat between the earth and the atmosphere. The routine observed data in the atmosphere do not permit a direct calculation of the diabatic heating by the above processes. But one should be able to estimate it indirectly from the physical laws governing the atmosphere. Wiin-Nielsen and Brown [32] used a closed system of equations, the thermodynamic equation and the vorticity equation, to solve for two unknowns: the vertical velocity and the diabatic heating. The other terms involved in the equations were computed from the observed data. Our attempt to compute the diabatic heating in the meridional plane is based on a similar procedure.

The thermodynamic equation in the mean state, in the same notation as before, is

$$\mathbf{v}\cdot\nabla\left(\frac{\partial\psi}{\partial p}\right)+\frac{\bar{\sigma}\omega}{f_0}=-\frac{R}{c_p p f_0}H. \qquad (34)$$

If we take the zonal average and rearrange the terms, equation (34) becomes

$$H_z=\frac{gp\ln\left(\frac{p_2}{p_1}\right)}{2\pi a^2\cos\phi(\Delta p)^2}\frac{\partial}{\partial\phi}[TH_{\Delta p}(\phi)]-\frac{c_p p\bar{\sigma}\omega_z}{R}. \qquad (35)$$

The writer (Vernekar [25]) has shown that the vertical velocity computed from (14) cannot be used to estimate the diabatic heating from the steady-state thermodynamic equation. ω_z used in evaluating H_z from (35) was, therefore, computed from (15) using $f[(uv)_z]$ as the only influencing function. It may be noted that this method of computing ω_z is not the same as that used by Wiin-Nielsen and Brown [32]. The appropriateness is to be verified *a posteriori*.

Before going into a discussion of the final results, it may be worthwhile to compare the preliminary results for a layer with those obtained by Brown [2]. He computed the diabatic heating on a daily basis to include the effect of standing and transient motion and then averaged over the whole month for a layer 800–400 mb. Our results

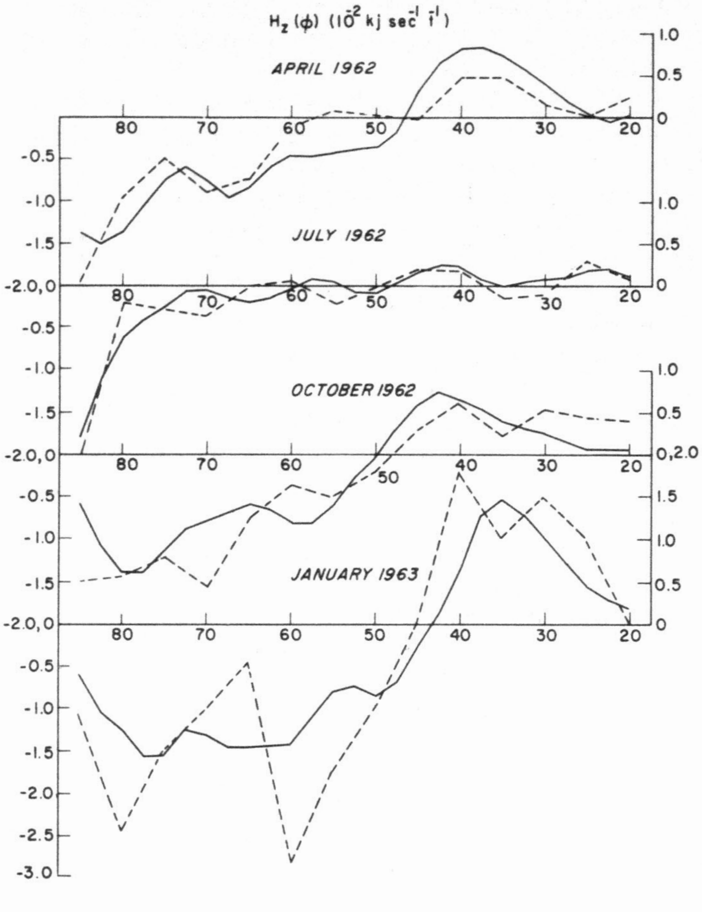

FIGURE 19.—The zonally veraged diabatic heating, H_z, as a function of latitude for the months of April, July, October 1962 and January 1963 in the unit: 10^{-2} kj. sec.$^{-1}$ t.$^{-1}$ Solid curves: H_z for the layer: 700–500 mb. computed in the present study. Dashed curve: H_z for the layer: 800–400 mb. according to Brown [2].

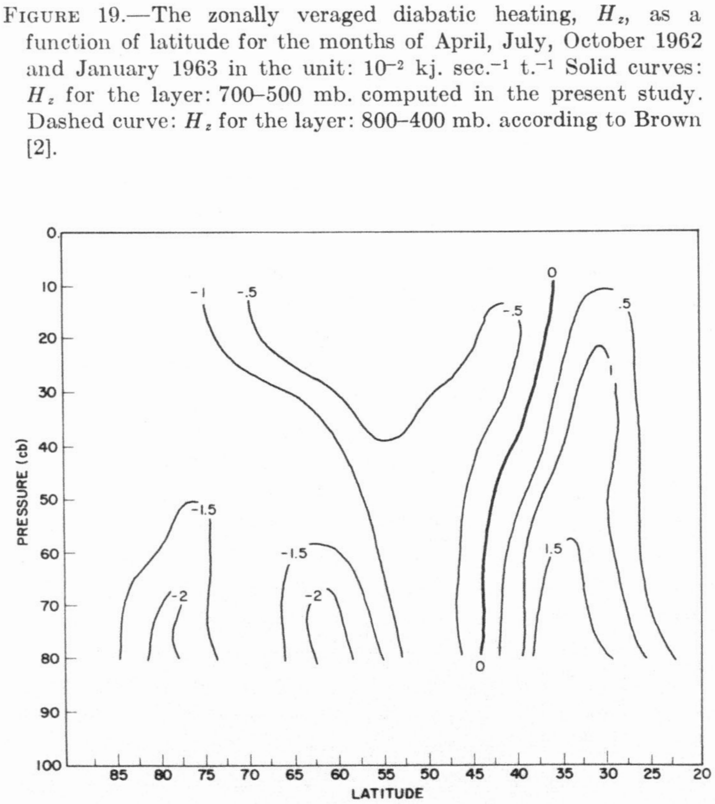

FIGURE 20.—The zonally averaged diabatic heating as a function of latitude and pressure for the month of January 1963 in the unit: 10^{-2} kj. sec.$^{-1}$ t.$^{-1}$

include only the effect of standing motion for the layer 700–500 mb. A comparison of $H_z(\phi)$ for April, July, October 1962 and January 1963, in the units 10^{-2} kj.t.$^{-1}$ sec.$^{-1}$, is shown in figure 19. The continuous curve shows our results while Brown's results are shown by a broken curve. Considering the difference in method of computations and the layer which they represent there is a striking similarity between the two sets of curves. The diabatic heating in the meridional plane for January 1963, in the units 10^{-2} kj.t.$^{-1}$ sec.$^{-1}$, is shown in figure 20. We shall compare the diabatic heating computed from the convergence of eddy transfer of sensible heat and the meridional circulations to the dominant heating factors in the atmosphere. The maximum heating occurs in the latitude belt 30–40°N. in the lower troposphere where the effect of the turbulent transfer of heat from the earth's surface and the latent heat of phase change of water can be the dominant factors. A gradual decrease in the upper troposphere is due to the influence of raditional cooling. North of 40°N. the net effect is a cooling. In the higher latitudes there are two distinct minima in the lower troposphere. One around 80°N. can be due to the ice cover over land and sea if the turbulent transfer of heat from the earth is the dominant factor. Another minimum around 60°N. as compared to the belt near 70°N. agrees with the heat balance of the earth by Budyko [3]. In the higher troposphere, the effect of the turbulent transfer of heat diminishes so that the cooling is only due to radiation.

5.3 THE GENERATION OF ZONAL AVAILABLE POTENTIAL ENERGY

The concept of available potential energy was originally introduced by Margules [12] and its mathematical definition was given by Lorenz [11]. The available potential energy is a measure of the amount of energy available for conversion into kinetic energy; it can be defined as the excess of total potential energy (potential and internal) above the amount which would exist if the isentropic surfaces were horizontal. The generation of available potential energy depends upon the mass integral of the product of the deviations of diabatic heating and temperature from the area average of these quantities. This definition of the generation of available potential energy, $G(A)$, can be expressed symbolically as:

$$G(A)=\frac{R}{gc_p}\int_0^{p_0}\int_s \frac{1}{\bar{\sigma}}\frac{1}{p}\alpha' H' ds\, dp. \tag{36}$$

Here the prime quantities are the deviations from their area average and s is the total area of the sphere. The generation of available potential energy is the covariance between temperature and heating. Thus available potential energy is produced when warm air is heated and cold air is cooled.

If we follow Lorenz, $G(A)$ can be considered as the sum of the zonal available potential energy, $G(A_z)$ and the eddy available potential energy $G(A_e)$. We shall be dealing here only with the contribution from the former for which the expression can be written as:

$$G(A_z)=\frac{R}{gc_p}\int_0^{p_0}\int_s \frac{1}{\bar{\sigma}}\frac{1}{p}\alpha_z' H_z' ds\, dp. \tag{37}$$

Using hydrostatic equilibrium, we can write

$$\alpha_z'=-\left[\frac{\partial\phi}{\partial p}\right]_z'. \tag{38}$$

If we substitute (38) in (37), the integral for $G(A_z)$, per unit area, over the latitude belt between 20°N. ($=\phi_1$) and 87.5°N. ($=\phi_2$) and for a layer of thickness Δp becomes

$$\frac{1}{s}G_{\Delta p}(A_z)=\frac{R}{c_p\bar{\sigma}p\,(\sin\phi_2-\sin\phi_1)}\int_{\phi_1}^{\phi_2} h_z' H_z' \cos\phi\, d\phi \tag{39}$$

where h_z' is the deviation from the area average of zonally averaged thickness (in meters) of the layer. $H_z(\phi, p)$ was obtained for January 1964 as a function of latitude and pressure as described in the previous section and h_z' was computed from the mean observed data. Hence the final results of $G(A_z)$ include only the effect of time-averaged motion. The integral in (39) was evaluated by Simpson's quadrature formula. $G(A_z)$ as a function of pressure, in the units 10^{-5} kj.m.$^{-2}$ sec.$^{-1}$ cb.$^{-1}$ is shown in the figure 21 and the contribution to $G(A_z)$ from each layer, in the units 10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ is given in table 1. The results show that the major portion of the production of the zonal available potential energy occurs in the lower troposphere. From the table it is clear that $4/5$ of the total generation of zonal available potential energy takes place in the lower half of the troposphere. $G(A_z)$ decreases rapidly in the upper troposphere and finally becomes negative in the layer 150–100 mb. The maximum production of zonal available potential energy in the lower troposphere is due to the fact that relatively warm air in the low latitudes is heated and that relatively cool air in the higher latitude is further cooled. In the higher troposphere the temperature gradient decreases. The tropopause level in the higher latitude is around 300 mb. above which the temperature gradient reverses the sign. The heating has the same distribution as in the lower troposphere. As a result generation of zonal available potential energy decreases. The layer 150–100 mb. lies in the stratosphere where the temperature and heating are out of phase by almost 180°; hence the covariance between them becomes negative. The negative value in this layer agrees with the study on the energetics of the lower stratosphere by Oort [16].

$G(A_z)$ for the entire atmosphere over the latitude belt 20°N. to 87.5°N. was 26.4×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, for January 1964. Wiin-Nielsen and Brown [32] estimated $G(A_z)$ to be 50.0×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ as the effect of standing as well as transient motion and 48.0×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ as the effect of standing motion, for January

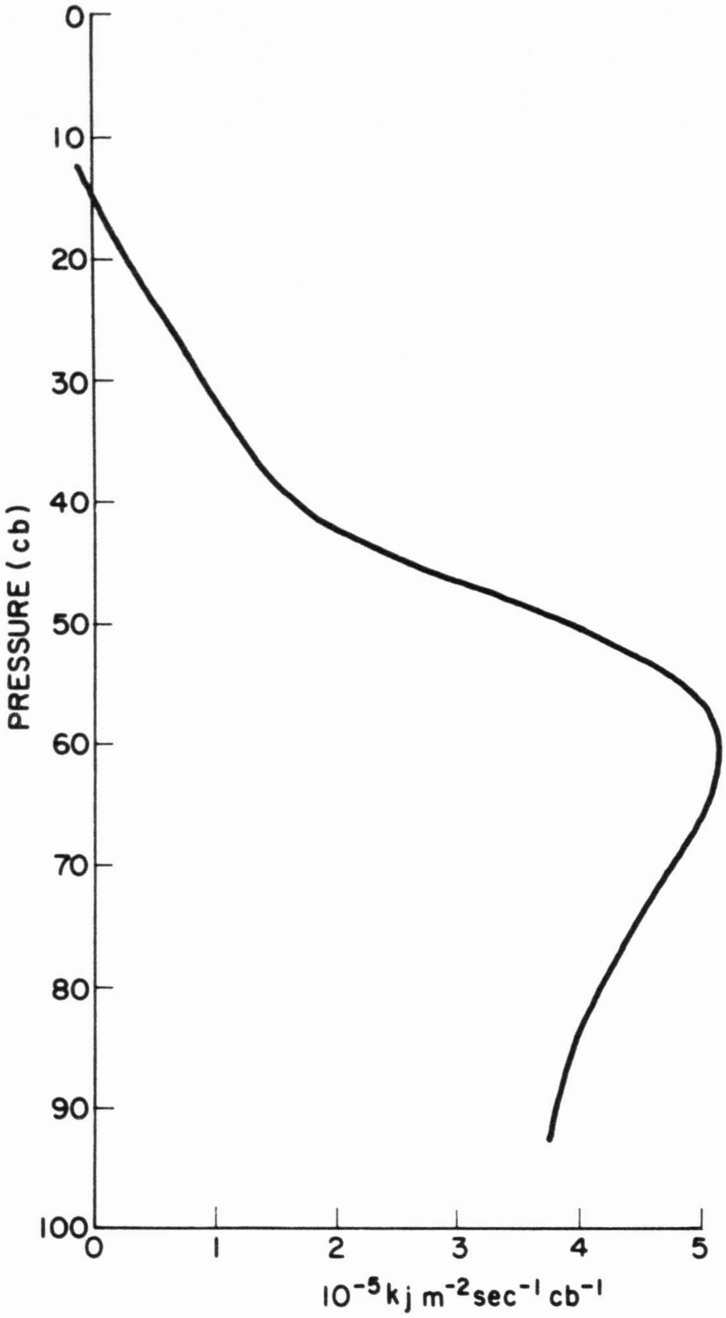

FIGURE 21.—The generation of zonal available potential energy as a function of pressure for the month of January 1964 in the unit: 10^{-5} kj.m.$^{-2}$ sec.$^{-1}$ cb.$^{-1}$

TABLE 1.—*The generation of zonal available potential energy for January 1964. Units: 10^{-4}kj.m.$^{-2}$ sec.$^{-1}$*

Layer (mb.)	1000–850	850–700	700–500	500–300	300–200	200–150	150–100	Total
$\frac{1}{s}G_{\Delta p}(A_z)$	5.65	6.46	10.28	3.32	0.62	0.06	−0.04	26.4

1959. From a comparison of the latter estimate with our results, it appears that our estimate differs approximately by a factor of two. Wiin-Nielsen and Brown computed $G(A_z)$ for the layer 800–400 mb. where it has its maximum value according to our results and the estimate for the entire troposphere (1000–200 mb.) was obtained assuming a constant distribution in the vertical. Hence their results are overestimated. If we make the same assumption using the value of $G(A_z)$ for the layer 700–500 mb. we obtain 51.4×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, for the entire atmosphere (1000–0 mb.) and 41.1×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$, for the troposphere (1000–200 mb.). Thus, these results are in very good agreement. Brown [2] computed $G(A_z)$ for January 1962 and 1963. His results are 36.1 and 39.4×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ for January 1962 and 1963 respectively. Hence, these results are also in close agreement. Phillips [18] obtained 21.3×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ from his two-level quasi-geostrophic model for general circulation, while Smagorinsky [22] obtained 22.1×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ from his primitive equation model for general circulation. These results are also in close agreement with those derived in this study.

5.4 CONVERSION OF THE ZONAL AVAILABLE POTENTIAL ENERGY TO THE ZONAL KINETIC ENERGY

It was first suggested by Margules [12] that the transformation process which produces kinetic energy in the atmosphere is the simultaneous rising of warm air and sinking of cold air. This transformation process can be mathematically defined as a mass integral of the product of the deviations of the vertical motion and temperature from their area average. Symbolically we may write

$$C(A, K)=-\frac{1}{g}\int_0^{p_0}\int_s \omega'\alpha' ds\, dp. \tag{40}$$

The contribution from the zonal flow to $C(A, K)$ in hydrostatic equilibrium is

$$C(A_z, K_z)=\frac{1}{g}\int_0^{p_0}\int_s \omega_z'\left[\frac{\partial\phi_z}{\partial p}\right]' ds\, dp$$

or

$$=-\frac{1}{g}\int_0^{p_0}\int_s \phi_z'\left[\frac{\partial\omega_z}{\partial p}\right]' ds\, dp. \tag{41}$$

In view of the equation of continuity, (41) becomes

$$C(A_z, K_z)=\frac{2\pi a}{g}\int_0^{p_0}\int_{-\pi/2}^{\pi/2} \phi_z' \frac{\partial}{\partial\phi}(v_z\cos\phi)d\phi\, dp \tag{42}$$

or

$$=\frac{2\pi a^2}{g}\int_0^{p_0}\int_{-\pi/2}^{\pi/2} f u_{gz} v_z \cos\phi\, d\phi\, dp, \tag{43}$$

where u_{gz} is the zonal average of west-east component of the geostrophic wind. The conversion per unit area over the region 20°N. and 87.5°N. in a layer of thickness Δp can be written as

$$\frac{1}{s}C_{\Delta p}(A_z, K_z)=\frac{\Delta p}{g\,(\sin\phi_2-\sin\phi_1)}\int_{\phi_1}^{\phi_2} f u_{gz} v_z \cos\phi\, d\phi. \tag{44}$$

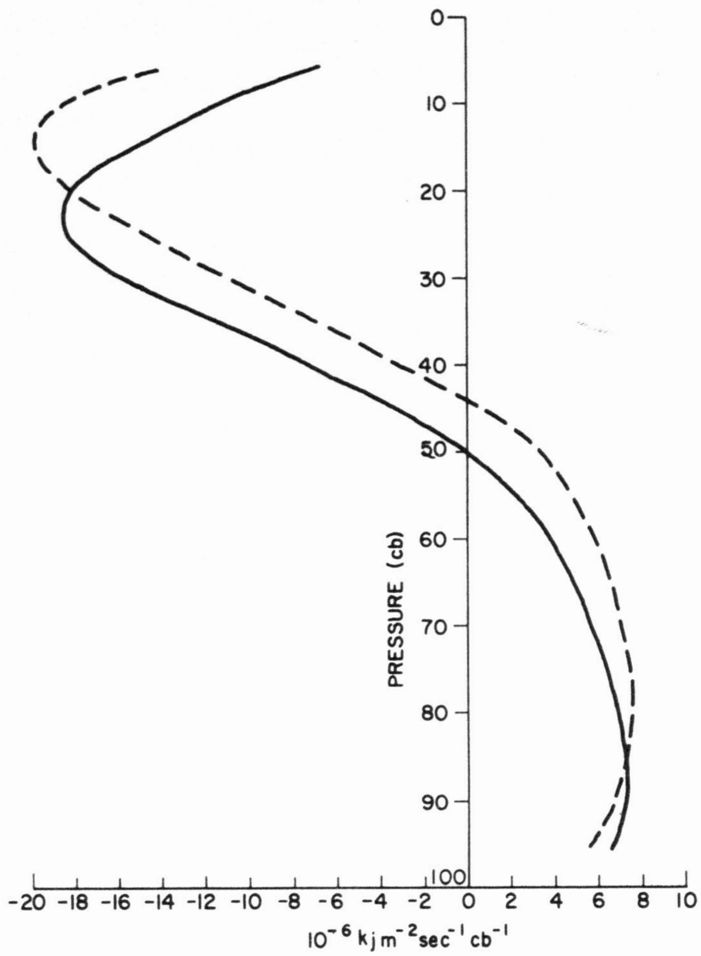

FIGURE 22.—The conversion between zonal available potential energy and zonal kinetic energy, $C(A_z, K_z)$, as a function of pressure for the month of January 1964 in the unit :10^{-6} kj.m.$^{-2}$ sec.$^{-1}$ cb.$^{-1}$ Solid curve: $C(A_z, K_z)$ due to the standing motion. Dashed curve: $C(A_z, K_z)$ due to the standing and transient motion.

From our results on mean meridional circulations it is possible to compute $C(A_z, K_z)$ from (41) as well as from (44). The latter equation was used since it permits a calculation for eight layers while the former only for seven layers. Computations were carried out on a daily basis to include the effect of transient and standing motion and also from mean monthly values to obtain the effect of standing motion. The results are illustrated in figure 22 as a function of pressure, in the units 10^{-6} kj.m.$^{-2}$ sec.$^{-1}$ cb.$^{-1}$ The broken curve shows the effect of transient and standing motion while the continuous curve refers to the standing motion alone. These results show that the effect of transient eddies is small. The interesting feature of the figure is that the zonal available potential energy is transformed into zonal kinetic energy in the lower troposphere where the baroclinicity is more pronounced and available potential energy is maximum while the reverse is true in the upper troposphere and lower stratosphere where the kinetic energy is maximum. Evidence that zonal kinetic energy is converted to zonal available potential energy in the stratosphere was indicated by White and Nolan [27] and Oort [16]. $C(A_z, K_z)$ for the entire atmosphere over the latitude belt 20°N. to 87.5°N. was -2.4×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ for January 1964. Wiin-Nielsen [30] obtained 1.03×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ for January 1959 over the same region. His computations were based on the vertical velocity at 600 mb. and the thickness of the layer: 850–500 mb. He further assumed a parabolic distribution in the vertical for the vertical velocity having zero value at 1000 mb. and 200 mb. $C(A_z, K_z)$ was thus estimated for the troposphere assuming a constant distribution in the vertical. If we make a similar assumption on the average value between 850–500 mb. we find 2.1×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ This value agrees fairly well with Wiin-Nielsen's results considering the period for which they represent. Saltzman and Fleisher [21] estimated 3.5×10^{-4} kj.m.$^{-2}$ sec.$^{-1}$ for the winter of 1959 using vertical velocities at 600 mb. and thickness for the 850–500-mb. layer over the Northern Hemisphere by extrapolating the data in the equatorial region. These results cannot be compared to our calculations because Palmén, Riehl, and Vuorela [17] showed that $C(A_z, K_z)$ is positive everywhere in the vertical over the tropical region. Jensen [8] presented data for the vertical velocity, computed by the adiabatic method, and temperature for different layers for January 1958. Wiin-Nielsen [31] used these data to evaluate the integral in (40) to show that these results should represent $C(A_z, K_z)-G(A_z)$ rather than $C(A_z, K_z)$. Since we have estimates of both $G(A_z)$ and $C(A_z, K_z)$ in the vertical such a comparison is shown in table 2. The conversion computed from Jensen's data is indicated by an asterisk in the table. Considering the differences in method of computation and the periods which they represent these results are not inconsistent with the conclusions drawn by Wiin-Nielsen.

6. CONCLUSIONS

6.1 SUMMARY

We have shown that the mean meridional circulations produced by eddy transports of heat and momentum in a quasi-geostrophic, adiabatic, and frictionless model are in agreement with those obtained by considering the balance of angular momentum in the atmosphere (Mintz and Lang [14] and Holopainen [7].

The magnitude of the mean meridional wind component, v_z, computed in the present study is less than 1 m. sec.$^{-1}$ The zonal average of the absolute value of the meridional

TABLE 2.—*A comparison between $C^*(A_z, K_z)$ and $C(A_z, K_z)-G(A_z)$. Units: 10^{-4} kj.m.$^{-2}$ sec.$^{-1}$*

Layer (mb.)	1000-850	850-700	700-500	500-300	300-200	200-100
$C^*(A_z, K_z)$	−10.53	−11.23	−8.04	−1.81	0.22	−0.23
$G(A_z, K_z)-G(A_z)$	−4.61	−5.34	−9.20	−3.92	−2.08	−2.01

wind component, $|v|_z$, is about 10 m. sec.$^{-1}$ On the average, therefore, v_z is at least one order of magnitude less than $|v|_z$. This justifies the assumption in the quasi-geostrophic theory that the mean meridional circulations are secondary processes.

Considering the separate effects of momentum and heat transports on the mean meridional circulations, it is found that the former is twice as effective as the latter, irrespective of the season. It may be concluded from this that the mean meridional circulations over the region north of 20°N. play a more important role in the angular momentum balance than in the heat balance.

The influence of the planetary scale motion on the mean meridional circulations during the winter months is much larger than the influence of the other scales of motion whereas the baroclinically unstable waves dominate the forcing mechanism during other seasons.

The seasonal variation of meridional circulations shows that the circulation cells move toward the Pole from winter to summer with a decrease in their intensity. A comparison between the intensities of the circulation for different seasons shows that the intensity in the summer is ⅓ of that in the winter. During the spring and fall the intensity of the circulation is almost the same and has a value which is about ⅔ that of the winter season. This oscillation of the meridional cells is closely related to the annual oscillation of the general circulation.

Net diabatic heating in the meridional plane is positive south of 40°N. and negative north of that latitude during winter months. In the upper troposphere heating decreases gradually with height in the region of net heating, whereas the cooling decreases sharply with height in the region of net cooling.

The generation of zonal available potential energy is maximum in the lower troposphere, where the baroclinicity is large, and decreases sharply in the upper troposphere and finally becomes negative in the lower stratosphere.

The conversion from zonal available potential energy to zonal kinetic energy occurs in the lower troposphere where the zonal available potential energy is maximum while the reverse process takes place in the upper troposphere where kinetic energy is maximum.

The present method of computing meridional circulations is sufficiently sensitive to give reasonable estimates on a daily basis.

6.2 CERTAIN CRITICAL REMARKS

Even if the results of this study agree with those of other investigators, we are not in a position to say that these are the true values of the mean meridional circulations for the real atmosphere. The effects of the diabatic heating and friction, on the mean meridional circulations, are by no means negligible as compared to the effects of eddy processes. This was revealed from a pilot study made for a single layer. The diabatic heating alone produced a single cell "Hadley type" circulation while friction has an effect similar to the eddy processes and produced a three-cell circulation.

The eddy transports of heat and momentum were computed from the objective height analysis obtained from the National Meteorological Center. Recently Holopainen [7] has pointed out that the momentum transports computed from the objective height analysis are overestimated as compared to those computed from the subjective analysis or from wind statistics. Such an overestimation of the momentum transports must have affected our results to some extent.

In our formulation of the problem, we have assumed that $f=f_0$ to be consistent with the quasi-geostrophic theory and to satisfy certain integral constraints (Wiin-Nielsen [28]). Such an assumption overestimates the meridional circulations in the polar region and underestimates them in the subtropical region.

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CORRECTION NOTICE

No. 7, July 1967, front cover, Contents, pp. 463–467: authors' names should be M. Wolk, F. Van Cleef, and G. Yamamoto.

No. 9, September 1967, p. 607, right column, third paragraph, first sentence should read "The basic wind data were taken from U.S. Navy Oceanographic Office Pilot Chart wind roses." Also fourth paragraph, first sentence should read ". . . U.S. Navy *Marine Climatic Atlas of the World*, . . . "

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